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INTEGRAL CRITERIA FOR THE BRITTLE STRENGTH OF CRACKED BODIES WITH DEFECTS IN THE PRESENCE OF VACANCIES AT THE TIP OF A CRACK. STRENGTH OF COMPACTED CERAMICS-TYPE BODIES

V. M. Kornev

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When referring to V. V. Novozhilov's work [1] researchers have mainly gave attention to the discrete (the author calls it sufficient) criterion of brittle strength (see [2–4] and the references in [3]). However, along with plausible arguments, Section 2 of [1] contains the integral (the author calls it necessary) criterion of brittle strength; application of this criterion in determining the breaking loads for bodies with angular notches has made it possible to eliminate some contradictions [5].

"While we can (by taking necessary precautions) grow a crystal without a single dislocation in a macroscopic volume, a crystal without point defects cannot be created in principle" [6 (p. 5) and 7]. The probability that defects appear both at the tip of a crack and in front of the crack increases considerably for the solid produced by powder compaction.

The present paper deals with discrete-integral criteria of brittle strength when atomic-lattice vacancies are present at the tip of a crack. If in front of such a crack there are atomic-lattice defects weakening interatomic interactions, the above integral criterion becomes a sufficient condition of the brittle strength of a solid. Defects of the type of vacancies in the vicinity of the crack tip are found to exert a profound effect on the critical lengths of cracks for various types of loading. Relations are obtained for the critical stress-intensity factors (SIF) of rupture cracks and for transverse and longitudinal shears when blunted (at the atomic level) cracks are considered. Simple relations are obtained which make it possible to determine the safe levels of rupture and shear loading for brittle cracked bodies via the critical SIF, the theoretical strengths of the weakest interatomic bond, and the characteristics of an atomic lattice with defects.

1. Defect in Front of Sharp Crack. Discrete-Integral Criteria of Brittle Strength. Cracks of normal rupture and of transverse and longitudinal shear in a simple Bravais lattice are studied. The tip of a crack of length $2l$ is schematically depicted in Fig. 1 with allowance for defects in the atomic lattice: the plane xOy is normal to the crack's rectilinear front Oz; the asterisks denote one or two vacancies, or one or two clusters of 2–4 vacancies at the crack tip (atom chains -2 and -1); the dots denote one or two vacancies or one or two clusters, of 2–4 vacancies before the crack tip (atom chains 1 and 2); and the dashed lines denote the crack's new front (the stability of this front is not discussed here). Atomic oscillations are ignored, i.e., the material is considered at temperature $T = 0$.

The probability of the formation of bi- and trivacancies is low in an ideal crystal [6]. It can be assumed that in bodies produced by powder compaction the concentration of various defects increases sharply. In addition, the presence of an impurity at the crack tip (foreigh atom in chain 0 in Fig. 1) attracts vacancies [7].

Following [1], we introduce integral criteria of brittle strength (a two-dimensional case, a crack of length $2l$):

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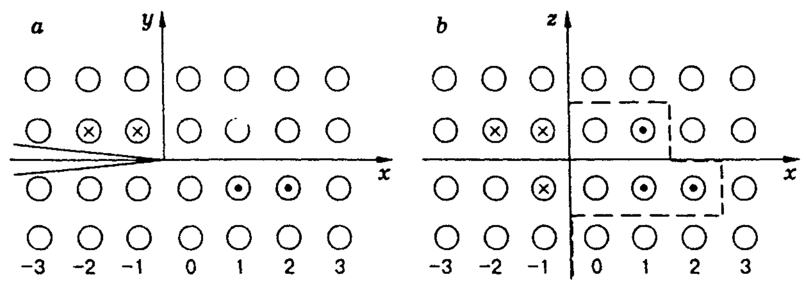

Fig. 1

— for sharp normal-rupture cracks [4]

$$\frac{1}{kr_e}\int_0^{nr_e}\sigma_y(x,0)dx \leqslant \sigma_m; \tag{1.1}$$

— for sharp transverse-shear cracks

$$\frac{1}{kr_e}\int_0^{nr_e}\tau_{xy}(x,0)dx \leqslant \tau_m; \tag{1.2}$$

— and for sharp longitudinal-shear cracks

$$\frac{1}{kr_e}\int_0^{nr_e}\tau_{yz}(x,0)dx \leqslant \tau_m. \tag{1.3}$$

Here σ_y, τ_{xy}, and τ_{yz} are the normal and shearing stresses at the crack tip (all have peculiarities of their own); $Oxyz$ is the rectangular coordinate system (the orientation of the axes relative to the right-hand part of the crack is shown in Fig. 1); r_e is the spacing between the atomic centers; σ_m and τ_m are the theoretical rupture and shear strengths, respectively; n and k are numbers (Fig. 2, $k < n$ and k is the number of active bonds); and nr_e is the interval of averaging in V. V. Novozhilov's formulas (1.1)–(1.3) (in Fig. 2 the notation for vacancies is the same as in Fig. 1). Figures 2a and 2b show schematically the location of defects near the crack tips for normal-rupture cracks in a simple Bravais lattice and for transverse-shear cracks in a compacted layer of atoms, respectively.

Criterion (1.1) fully coincides in form with the criterion of [4], but in the former, due to the presence of vacancies at the crack tip, use should be made of some classical representation of the solution, because, within the crack, the interatomic-interaction forces are either negligibly small or absent. In the presence of vacancies at the tip of a crack no lattice capture occurs, and hence it is not discussed here. Using the solution for stresses on the extension of a sharp crack $y = 0$ via the SIF $K_{\rm I}$, $K_{\rm II}$, and $K_{\rm III}$, we can write

$$\sigma_y(x,0) = \sigma_\infty + \frac{K_{\rm I}}{(2\pi x)^{1/2}}, \quad \tau_{xy}(x,0) = \tau_\infty + \frac{K_{\rm II}}{(2\pi x)^{1/2}}, \quad \tau_{yz}(x,0) = \tau_\infty + \frac{K_{\rm III}}{(2\pi x)^{1/2}},$$

where σ_∞ and τ_∞ are the characteristic stresses given either at infinity or on some contour of a finite solid.

Without vacancies before the crack front, i.e., for $k = n = 1$, the suggested criteria (1.1)–(1.3) become necessary conditions of strength (see [1]).

Upon appropriate transformations we obtain estimates for the critical SIF $K_{\rm I}^{*0}$, $K_{\rm II}^{*0}$, and $K_{\rm III}^{*0}$. For a sharp normal-rupture crack in the presence of vacancies at the crack tip, we have

$$\frac{K_{\rm I}^{*0}}{\sigma_\infty} \leqslant \left(\frac{\sigma_m}{\sigma_\infty}\frac{k}{n} - 1\right)\left(\frac{\pi n r_e}{2}\right)^{1/2}, \tag{1.4}$$

and for sharp transverse- and longitudinal-shear cracks, in relation (1.4) $K_{\rm I}^{*0}$ is replaced by $K_{\rm II}^{*0}$ and $K_{\rm III}^{*0}$, σ_m is replaced by τ_m, and σ_∞ by τ_∞. Thus, the three brittle-strength criteria have structures which coincide

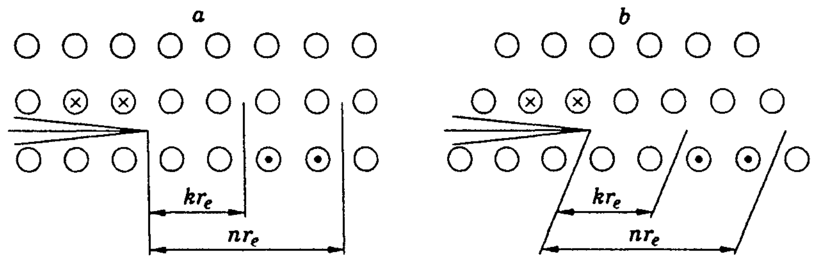

Fig. 2

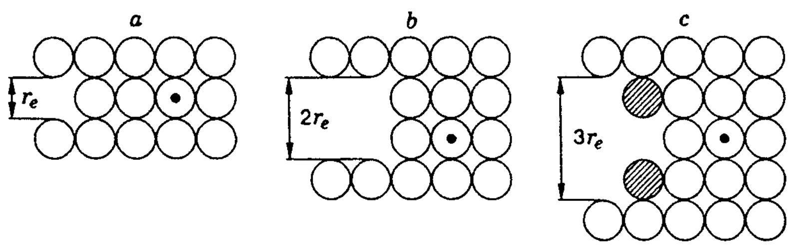

Fig. 3

with accuracy to renotation when sharp cracks are considered: the parameters n and k, which characterize the range of averaging and the number of active interatomic bonds, appear identically in the formulas for the critical SIF $K_{\rm I}^{*0}$, $K_{\rm II}^{*0}$, and $K_{\rm III}^{*0}$.

Relations (1.1)–(1.4) contain the quantities σ_m and τ_m which describe the theoretical strengths. As a rule, however, "... calculations of ideal strengths were all performed under the assumption of a homogeneous deformation. Strictly speaking, the results should be applicable to such problems of heterogeneous deformation in which stress and deformation change insignificantly at distances comparable with the range of action of interatomic forces. This is not true for the tip of a crack..." [8, p. 98]. Attempts to estimate the rupture strength of atomic chains are reported in [3, 9].

It is reasonable to use criteria of the (1.4) type only when vacancies or a foreign atom with a very weak bond are present at the tip of a crack. The presence of vacancies or a cluster of vacancies at the tip of a crack leads to crack blunting at the atomic level.

2. Defect in Front of a Blunted Crack and Discrete-Integral Criteria of Brittle Strength. We consider normal-rupture, transverse- and longitudinal-shear cracks that are blunted at the atomic level. Let the atomic structure of a material correspond to a simple Bravais lattice. Figure 3 shows schematically three simplest situations (the dots denote vacancies in front of a flat crack-like cavity). In Fig. 3c, one or both dashed atoms can be absent. For relations (1.1)–(1.3), we have $n = 3$ and $k = 2$ (Fig. 3a), and $n = 2$ and $k = 1$ (Figs. 3b and 3c). A similar situation also arises in a closely-packed atomic layer. For a narrow flat notch, the stress distributions near the notch apex can be expressed via the SIF for a crack into which the notch transforms as the curvature radius ρ at the notch apex approaches zero, i.e., $\rho \to 0$.

Figure 4 shows the location of the rectangular xOy and the polar $Or\theta$ coordinate systems relative to the notch apex. In these coordinate systems, three stress distributions relative to the notch axis were obtained in [10, 11]:

Symmetrical

$$\sigma_x = \frac{K_{\rm I}}{(2\pi r)^{1/2}} \left\{ \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3}{2}\theta \right) - \frac{\rho}{2r} \cos \frac{3}{2}\theta \right\},$$

$$\sigma_y = \frac{K_{\rm I}}{(2\pi r)^{1/2}} \left\{ \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3}{2}\theta \right) + \frac{\rho}{2r} \cos \frac{3}{2}\theta \right\}, \tag{2.1}$$

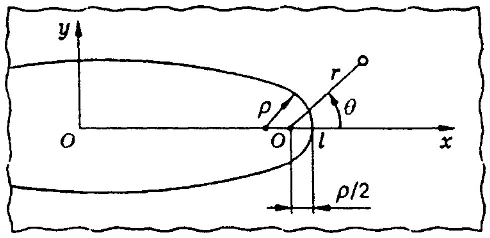

Fig. 4

$$\tau_{xy} = \frac{K_{\rm I}}{(2\pi r)^{1/2}} \left\{ \cos\frac{\theta}{2} \sin\frac{\theta}{2} \cos\frac{3}{2}\theta - \frac{\rho}{2r} \sin\frac{3}{2}\theta \right\};$$

Asymmetrical

$$\sigma_x = -\frac{K_{\rm II}}{(2\pi r)^{1/2}} \left\{ \sin\frac{\theta}{2} \left(2 + \cos\frac{\theta}{2} \cos\frac{3}{2}\theta\right) + \frac{\rho}{2r} \sin\frac{3}{2}\theta \right\},$$

$$\sigma_y = \frac{K_{\rm II}}{(2\pi r)^{1/2}} \left\{ \sin\frac{\theta}{2} \cos\frac{\theta}{2} \cos\frac{3}{2}\theta - \frac{\rho}{2r} \sin\frac{3}{2}\theta \right\}, \tag{2.2}$$

$$\tau_{xy} = \frac{K_{\rm II}}{(2\pi r)^{1/2}} \left\{ \cos\frac{\theta}{2} \left(1 - \sin\frac{\theta}{2} \sin\frac{3}{2}\theta\right) - \frac{\rho}{2r} \cos\frac{3}{2}\theta \right\};$$

For longitudinal shear

$$\tau_{xz} = -\frac{K_{\rm III}}{(2\pi r)^{1/2}} \sin\frac{\theta}{2}, \quad \tau_{yz} = \frac{K_{\rm III}}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}. \tag{2.3}$$

The three stress fields (2.1)–(2.2) in the limit for $\rho \to 0$ pass into the stress fields for rupture, transverse, and longitudinal shears, crack respectively. Note that relations (2.1)–(2.3) (see [10, 11]) do not contain terms defined by σ_∞ and τ_∞.

The structure of relations (2.1)–(2.3) in each of the three cases under consideration permits a limit passage for $\rho \to 0$. However, for all types of cracks blunted at the atomic level with the least curvatures at the notch apex, one can consider only the series $\rho \cong 0.5r_e$, $1.0r_e$, $1.5r_e, \ldots, r_e i/2, \ldots$ for $i \geqslant 1$ (i is the number of the missing atomic layers, $i = 1$, 2, and 3 in Figs. 3a–3c). For a finite ρ at the notch apex, i.e., for $\theta = 0$ and $r = \rho/2$ on its surface, we have [see (2.1)–(2.3)]: for the first case $\sigma_x = 0$ and $\tau_{xy} = 0$; for the second $\sigma_x = 0$ and $\sigma_y = 0$; and for the third $\tau_{xy} = 0$. One can, therefore, readily understand the representation of the brittle strength criteria (1.1)–(1.3) in which σ_y is used in the first case [1], τ_{xy} in the second, and τ_{yz} in the third.

Since the sufficiently detailed data on the critical lengths of normal-rupture cracks were previously obtained in [3, 4] using V. V. Novozhilov's modified discrete criterion for brittle strength, we now consider, for comparison, the problem of plane extension in the presence of a crack or a notch. If we take into account the location of the coordinate axes relative to the flat notch (Fig. 4), the integral brittle-strength criterion for a blunt normal-rupture crack has the form (nr_e is the interval of averaging)

$$\frac{1}{kr_e} \int\limits_{\rho/2}^{\rho/2+nr_e} \left[\sigma_\infty + \frac{K_{\rm I}}{(2\pi x)^{1/2}} \left(1 + \frac{\rho}{2x}\right)\right] dx \leqslant \sigma_m.$$

For this problem we can determine the SIF $K_{\rm I}$ via the specified stresses σ_∞ ($y \to \infty$) and the length of a blunt crack or a flat narrow notch $2l_{nk} = 2l(n, k)$ as follows: $K_{\rm I} = \sigma_\infty(\pi l_{nk})^{1/2}$, where the integral subscripts n and k characterize the interval of averaging and the number of active bonds.

TABLE 1

σ_m/σ_∞	n = 1	2	3	4	Griffith's critical cracks
	$2l^*_{n1}/r_e$				
5	(16) —	5 —			
10	(81) 530	32 470	17 450	9 440	430
20	(361) 2120	162 1880	97 1800	64 1760	1720
100	(9801) 53110	4802 47025	3137 44970	2304 44000	43000

After appropriate manipulations for a blunt normal-rupture crack (notch) we obtain the critical SIF $K_{\rm I}^*$, which is expressed in terms of the SIF $K_{\rm I}^{*0}$ for a sharp crack as

$$K_{\rm I}^* = K_{\rm I}^{*0}\Big(\frac{\rho}{2nr_e}+1\Big)^{1/2} \tag{2.4}$$

and the critical length $2l^*_{nk}$ of this blunt crack

$$\frac{2l^*_{nk}}{r_e} = \Big(\frac{\sigma_m}{\sigma_\infty}-\frac{n}{k}\Big)^2\frac{k^2}{n}\Big(\frac{\rho}{2nr_e}+1\Big). \tag{2.5}$$

The last parentheses in relations (2.4) and (2.5) are the corrections for crack blunting at the atomic level. In the particular case in which $n = k = 1$, relation (2.5), after transformations, coincides with the relation from Section 3 of [1]. An essential difference between relations (2.4) and (2.5) and the relations of [1, 4] is that the latter contain information on the notch curvature radius in the apex [1] and on the defects in front of a blunt crack and their location [4].

The parameter $2l^*_{nk}/r_e$ for a sharp and a blunt crack were calculated by formula (2.5). Table 1 gives the calculation results and a comparison between the relative critical lengths $2l^*_{n1}/r_e$ of sharp normal-rupture cracks ($\rho = 0$) in the presence of vacancies at the tip of the crack and in front of it ($n \geqslant 2$) when there is only one active interatomic bond ($k = 1$) and the interval of averaging is varied. The parameter $n = 1$, 2, 3, and 4 characterizes this interval. In addition, for a given level of loading σ_m/σ_∞, the relative critical lengths $2l^*_{n1}/r_e$ are presented, which were obtained from relation (2.5) by the integral criterion of brittle-strength (the upper line) and in [4] by the discrete criterion of brittle strength (the lower line; recall that in [4] use was made of the Morse potential for the Griffith crack as well).

Note that for $n = k = 1$ (i.e., there is no vacancy in front of a crack), the integral criterion of brittle strength (2.5) is only a necessary criterion [1] and, therefore, even for an overstrained zero bond (see Fig. 1) an atomic system without defects continues to withstand an increasing load (the fictitious critical lengths $2l^*_{11}/r_e$ are enclosed in the parentheses). The critical lengths $2l^*_{n1}/r_e$ that were determined by the integral criterion (2.5) for $n = 2$, 3, and 4 when vacancies exist at the tip of the crack, and in front of it, differ by an order of magnitude from the critical lengths $2l^*_{n1}/r_e$ ($n = 1$, 2, 3, and 4) that were obtained by the discrete criterion and from the critical length of the Griffith crack. The critical lengths of cracks were found to be highly sensitive to the appearance of vacancies simultaneously at the tip and in front of cracks.

Table 2 shows the results of calculation of the critical lengths $2l^*_{n1}/r_e$ of blunt normal-rupture cracks using only the integral brittle-strength criterion in the absence ($n = 1$) and in the presence of vacancies in front of the notch ($n \geqslant 2$) when there is a single active bond ($k = 1$); the relative curvature radius ρ/r_e at the notch apex for each level of loading is also given.

TABLE 2

σ_m/σ_∞	ρ/r_e	n			
		1	2	3	4
		$2l^*_{n1}/r_e$			
	0.5	(20)	11	5	—
5	1.0	(24)	12	5	—
	1.5	(28)	13	5	—
	0.5	(102)	36	18	10
10	1.0	(122)	40	20	11
	1.5	(142)	44	21	11
	0.5	(452)	183	105	68
20	1.0	(542)	203	113	72
	1.5	(632)	223	121	76
	0.5	(12252)	5403	3398	2448
100	1.0	(14702)	6003	3660	2592
	1.5	(17152)	6603	3921	2736

Comparison of the calculation results (see Tables 1 and 2) for the critical lengths of sharp and blunted cracks leads to the conclusion that the integral brittle-strength criterion (2.5) responds weakly to changes in crack blunting at the atomic level. In the presence of vacancies in front of a notch ($k = 1$ and $n = 2$, 3, and 4), an overstrained interatomic bond is broken to yield a notch with a sharp crack; the stability of the resulting, more complicated system has yet to be studied. In the absence of vacancies in front of the notch ($n = k = 1$), but in the presence of the curvature radius at the notch apex ($\rho/r_e = 0.5$, 1, and 1.5), fictitious critical lengths (enclosed in the parentheses) that agree with those for the necessary brittle-strength criterion (see Table 1) were obtained.

Figure 5 illustrates the effect of the interval of averaging on the parameters of the relative critical lengths $2l_{nk}/r_e$ for a given level of loading $\sigma_m/\sigma_\infty = 10$: curve 1 is constructed for $k = 1$ and $\rho/r_e = 0$; 2 for $k = 1$ and $\rho/r_e = 0.5$; 3 for $k = 1$ and $\rho/r_e = 1$; 4 for $k = 2$ and $\rho/r_e = 0$; 5 for $k = 2$ and $\rho/r_e = 0.5$; and 6 for $k = 2$ and $\rho/r_e = 0.5$. Each curve consists of two parts: a dashed part and a solid part. The solid parts of the curves correspond to the actual critical lengths $2l^*_{nk}/r_e$ and the dashed ones correspond to the fictitious critical lengths, which are determined by the necessary strength criteria [1] since $n = k$. Crack blunting at the atomic level exerts a weak effect on the critical-length parameter $2l^*_{nk}/r_e$ when $n > k$.

We consider the other types of blunt cracks. Using relations (2.2) and (2.3) and taking into account the terms τ_∞, we obtain the critical SIF for a blunt crack of transverse $K^*_{\rm II}$ and longitudinal $K^*_{\rm III}$ shears (the critical SIF of sharp cracks coincide: $K^{*0}_{\rm II} = K^{*0}_{\rm III}$):

$$K^*_{\rm II} = K^{*0}_{\rm II}\Big(\frac{\rho}{2nr_e} + 1\Big)^{1/2}\Big[\Big(\frac{\rho}{2nr_e} + 1\Big)^{1/2} + \Big(\frac{\rho}{2nr_e}\Big)^{1/2}\Big]^2; \qquad (2.6)$$

$$K^*_{\rm III} = K^{*0}_{\rm III}\Big[\Big(\frac{\rho}{2nr_e} + 1\Big)^{1/2} + \Big(\frac{\rho}{2nr_e}\Big)^{1/2}\Big]. \qquad (2.7)$$

It should be emphasized that if the structures of the relations of the critical SIF $K^{*0}_{\rm I}$, $K^{*0}_{\rm II}$, and $K^{*0}_{\rm III}$ of the three types of sharp cracks are the same [see (1.4)], the critical SIF $K^*_{\rm I}$, $K^*_{\rm II}$, and $K^*_{\rm III}$ of blunt cracks differ considerably [see (2.4), (2.6), and (2.7)]. The reduced curvature parameter $\rho/2nr_e$ at the notch apex appears in relations (2.4), (2.6), and (2.7) differently and is connected with the interval of averaging.

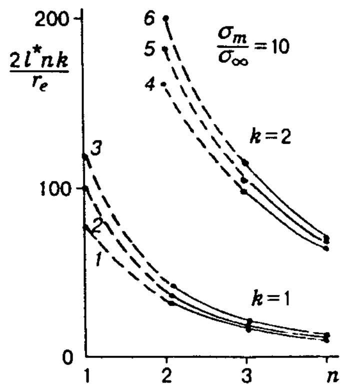

Fig. 5

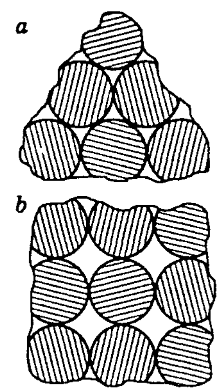

Fig. 6

3. Estimation of the Brittle Strength of Compacted Materials. We shall use the critical SIF obtained for different types of sharp and blunt cracks to estimate the attainable brittle strength of solids with defects compared with the theoretical strength of interatomic bonds. We shall further use the critical SIF $K_{\rm I}^{*0}$, $K_{\rm II}^{*0}$, and $K_{\rm III}^{*0}$ for sharp cracks [see (1.4)], because the critical SIF $K_{\rm I}^*$, $K_{\rm II}^*$, and $K_{\rm III}^*$ of blunt cracks depend only slightly on crack blunting at the atomic level [see (2.4)–(2.6)]. We assume that the material is produced by static or dynamic compaction of ideal cylinders with equal radii. Let the remaining pores be unfilled.

The cross section that is normal to the axis of cylinders produced from a porous material is presented in Fig. 6, where (a) corresponds to a close packing of cylinders and (b) to a packing of regular cylindrical particles whose centers form a simple Bravais lattice. In both cases the infinite surface is weakened by hypocyclic holes with three apices and astroid-shaped holes.

We consider a plane weakened by a single hypocyclic hole with a small number of apices; for this case we have the following estimates of the maximum SIF [11, pp. 209 and 408–409]:

$$K_{\rm I} \cong (0.94 \text{ to } 1.08)\sigma_\infty(\pi a)^{1/2}, \quad K_{\rm II} \cong (0.94 \text{ to } 0.64)\tau_\infty(\pi a)^{1/2}, \quad K_{\rm III} \cong (1 \text{ to } 0.86)\tau_\infty(\pi a)^{1/2}. \quad (3.1)$$

Here a is the radius of a circumscribed circle for a hypocycloid and an astroid; and σ_∞ and τ_∞ are the uniaxial tensile and shear stresses given at infinity. It is obvious that one can approximate the first and the third relations from (3.1) accurately enough and the second one comparatively roughly, by the expressions below (under the same loading conditions)

$$K_{\rm I} = \sigma_\infty(\pi l)^{1/2}, \quad K_{\rm II} = \tau_\infty(\pi l)^{1/2}, \quad K_{\rm III} = \tau_\infty(\pi l)^{1/2}, \tag{3.2}$$

where $2l$ is the length of an equivalent sharp crack.

We consider the extension of a half-plane with an edge curvilinear notch with a turning point on the contour [11, p. 147], which simulates the extension of a half-plane obtained by compaction of the cylinders. In this case the SIF are given by the expression

$$K_{\rm I} = (3 \cdot 2^{1/2}/4)\sigma_\infty(\pi a)^{1/2} \tag{3.3}$$

(a is the notch depth), which agrees with the SIF of the edge crack [11, p. 111]

$$K_{\rm I} = 1.12\sigma_\infty(\pi l)^{1/2}, \quad K_{\rm II} = 1.12\tau_\infty(\pi l)^{1/2} \tag{3.4}$$

(l is the length of an equivalent edge crack).

In essence, acute-angled defects are replaced by cracks. The greatest SIF are obtained for a plane with an astroid [the first relation from (3.1)], and, for a half-plane, the greatest SIF are obtained when there is a sharp notch (3.3) or an edge crack (3.4).

Since acute-angled defects in a solid are located in a definite order, we present estimates of the SIF of periodically located equivalent cracks in a plane and in a half-plane. The greatest SIF $K_{\rm I}$, $K_{\rm II}$, and $K_{\rm III}$ are obtained for in a collinear system of cracks and for a doubly periodic crack system, which weakens the plane, and, moreover, $K_{\rm I} \cong 2\sigma_\infty l^{1/2}$ and $K_{\rm II} \cong K_{\rm III} \cong 2\tau_\infty l^{1/2}$. For a half-plane, the greatest SIF $K_{\rm I}$ is obtained for an edge crack, and the greatest SIF $K_{\rm II}$ for a periodic system of edge cracks, with $K_{\rm I} \cong 2\sigma_\infty l^{1/2}$ and $K_{\rm II} \cong 2\tau_\infty l^{1/2}$. Thus, the final estimates of the maximum SIF can be written as

$$K_{\rm I} \cong 2\sigma_\infty l^{1/2}, \quad K_{\rm II} \cong K_{\rm III} \cong 2\tau_\infty l^{1/2}. \tag{3.5}$$

Let us use relation (1.4) for the critical SIF of sharp cracks of the three types. After the appropriate transformations we obtain a relation that characterizes the attainable level of tensile and shear strengths of cracked compacted solids in the presence of vacancies at the tip of a crack and in front of it:

$$\frac{\sigma_\infty}{\sigma_m} = \frac{k}{n}\left[\left(\frac{2}{\pi n}\right)^{1/2}\frac{K_{\rm I}^{*0}}{\sigma_\infty (r_e)^{1/2}} + 1\right]^{-1}. \tag{3.6}$$

Here σ_m are stresses that correspond to the theoretical breaking strength of the weakest interatomic bond in the system under consideration, and the other designations are the same as in Section 1.

The relations that characterize the shear strength of cracked compacted solids have the form of (3.6) with the difference that $K_{\rm I}^{*0}$ is replaced by $K_{\rm II}^{*0}$ or $K_{\rm III}^{*0}$, σ_∞ by τ_∞, and σ_m by τ_m (τ_m are stresses that correspond to the theoretical shear strength of the weakest interatomic bond).

If we ignore minor terms for cracks that are not very short at the atomic level $[(l_*/r_e)^{1/2} \gg 1]$, we obtain simple estimates of the relative critical lengths of normal-rupture cracks from (3.5) and (3.6) which have the form

$$\frac{l_*}{r_e} \cong \frac{8}{\pi}\frac{k^2}{n}\left(\frac{\sigma_m}{\sigma_\infty}\right)^2. \tag{3.7}$$

For transverse- and longitudinal-shear cracks, σ_m/σ_∞ is replaced by τ_m/τ_∞ in relation (3.7).

The relative critical lengths l_*/r_e (for the given atomic-lattice parameter r_e) depend strongly on the number k of active interatomic bonds at the tip of a crack and on the theoretical strength σ_m of the weakest interatomic bond, and comparatively weakly on the averaging interval nr_e, i.e., on the parameter n. Recall that the theoretical strength σ_m is dependent not only on the interatomic potential, but also on the number of interatomic bonds per unit area, i.e., on r_e. Therefore, to increase the strength of cracked compacted bodies, it is desirable: 1) to improve the quality of compaction by static or dynamic compression, sometimes involving subsequent caking (the quality of compaction in the chosen model is described by the parameter k^2/n; the actual values of this parameter are from 1/2 to 4/3) and 2) to exclude admixtures with small theoretical strength σ_m.

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